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Local and global deadlock-detection in component-based systems are NP-hard

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Abstract

Interaction systems are a formal model for component-based systems. Combining components via connectors to form more complex systems may give rise to deadlock situations. We present here a polynomial time reduction from 3-SAT to the question whether an interaction system contains deadlocks.

Keywords: Computational complexity; Concurrency; Safety/security in digital systems

1. Introduction

We consider a setting where components are combined via connectors to form more complex systems, see, e.g., [6,10,11] and [1]. Each individual component i offers ports $a_i, b_i, \ldots \in A_i$ for cooperation with other components. Each port in A_i represents an action of component i. The behavior of a component can be represented via a labeled transition system with starting state, where in each state there is at least one action available. Components are glued together via connectors, where each connector interlinks certain ports. In the global system obtained by gluing components local deadlocks may arise where a group of components is engaged in a cyclic waiting and will thus no longer participate in the progress of the global system (cf. [12]). If all components are involved, we speak of a global deadlock. We show here that detecting either kind of deadlock is NP-hard by (polynomially) encoding the classic 3-SAT problem in deadlock detection for interaction systems. For this we show two things: First, we show that in any system constructed for a formula there is a local deadlock iff there is a global deadlock. Second, we show that a formula is satisfiable iff there is a reachable global deadlock in the corresponding system. To ensure these properties, we introduce components for a clause of a 3-CNF formula, which will always be able to progress while the clause evaluates to false. So at the time a deadlock occurs no progress is possible and, that is, no clause evaluates to false. The paper is organized as follows. Section 2 contains the basic definitions. Section 3 gives the polynomial time reduction from 3-SAT to the problem of deadlock detection in interaction systems. Section 4 contains a short conclusion and a discussion of related work.

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2. Components, connectors and interaction systems

We consider here *interaction systems*, a model for component-based systems that was proposed and discussed in detail in [6,11,7] and [1]. An *interaction system* is a tuple $Sys = (K, \{A_i\}_{i\in K}, C, \{T_i\}_{i\in K})$,[1] where K is the set of *components*. Without loss of generality, we assume $K = \{1,\dots,n\}$. Each component $i \in K$ offers a finite set of *ports* A_i for cooperation with other components. The port sets A_i are pairwise disjoint. Cooperation is described by *connectors*. A *connector* is a set of actions $c \subseteq \bigcup_{i\in K} A_i$, where for each component i at most one action $a_i \in A_i$ is in c. A connector set C is a set of connectors, s.t. every action of every component occurs in at least one connector of C and no connector contains any other connector.

The local behavior of each component i is described by $T_i = (Q_i, A_i, \rightarrow_i, q_i^0)$, where Q_i is the finite set of local states, $\rightarrow_i \subseteq Q_i \times A_i \times Q_i$ the local transition relation and $q_i^0 \in Q_i$ is the local starting state. Given a connector $c \in C$ and a component $i \in K$ we denote by $i(c) := A_i \cap c$ the participation of i in c.

For $q_i \in Q_i$ we define the set of *enabled actions* $ea(q_i) := \{a \in A_i \mid \exists q_i' \in Q_i, \text{ s.t. } q_i \xrightarrow{a} q_i'\}$. We assume that the T_i's are non-terminating, i.e., $\forall i \in K\ \forall q_i \in Q_i$, $ea(q_i) \neq \emptyset$.

The global behavior $T_{Sys} = (Q, C, \rightarrow, q^0)$ of Sys (henceforth called *global transition system*) is obtained from the behaviors of the individual components, given by the transition systems T_i, and the connectors C in a straightforward manner:

- $Q = \prod_{i\in K} Q_i$, the Cartesian product of the Q_i, which we consider to be order independent. We denote states by tuples $(q_1,\dots,q_n)$ and call them global states.
- the relation $\rightarrow \subseteq Q \times C \times Q$, defined by

$$\forall c \in C\ \forall q, q' \in Q,$$
$$q = (q_1,\dots,q_n) \xrightarrow{c} q' = (q_1',\dots,q_n') \quad \text{iff}$$
$$\forall i \in K\ \big(q_i \xrightarrow{i(c)}{}_i q_i' \text{ if } i(c) \neq \emptyset \text{ and } q_i' = q_i \text{ otherwise}\big).$$

- $q^0 = (q_1^0,\dots,q_n^0)$ is the starting state for Sys.

In the global transition system a transition labeled c may take place when each component participating in c is ready to perform $i(c)$.

For an example of an interaction system see Example 1 at the end of Section 3.

For a global state $q = (q_1,\dots,q_n) \in Q$ we refer to the local state q_j of component $j \in K$ by $q(j)$.

Let $q = (q_1,\dots,q_n) \in Q$ be a global state. We say that some non-empty set $D = \{j_1, j_2, \dots, j_{|D|}\} \subseteq K$ of components is in *deadlock* in q if $\forall i \in D\ \forall c \in C$, s.t. $c \cap ea(q_i) \neq \emptyset\ \exists j \in D$, s.t. $j(c) \not\subseteq ea(q_j)$. We say that i waits for j then.

A system has a local *deadlock* in some global state q if there is $D \subseteq K$, that is in deadlock in q. If $D = K$, the system is globally deadlocked. Hence a global deadlock is a special case of a local deadlock. A system is *deadlock-free*, if there is no reachable state q and $D \subseteq K$, such that D is in deadlock in q.

We denote by IS the set of all interaction systems and by LDIS (GDIS) the set of interaction systems that contain local (global) deadlocks:

$$LDIS := \{Sys \in IS \mid Sys \text{ contains reachable local deadlocks}\},$$
$$GDIS := \{Sys \in IS \mid Sys \text{ contains reachable global deadlocks}\}.$$

We consider the well-studied NP-complete 3-SAT problem [5,4] where the formula is a conjunction of clauses k_i, each of which is a disjunction of 3 literals (i.e., possibly negated variables) and reduce it to *LDIS* as well as *GDIS*.

3. Reducing 3-SAT to LDIS and GDIS

Let $F = k_1 \wedge \dots \wedge k_n$ with $k_i = (l_{(i,1)} \vee l_{(i,2)} \vee l_{(i,3)})$ be a propositional formula in 3-CNF, where $l_{(i,1)}, l_{(i,2)}, l_{(i,3)}$ are positive literals (i.e., variables) or negative literals (i.e., negated variables). In the following, we construct an interaction system $Sys(F)$, s.t. $(F \in 3\text{-}SAT) \Leftrightarrow (Sys(F) \in GDIS) \Leftrightarrow (Sys(F) \in LDIS)$. We represent each clause k_i by a component $(i,0)$ and each literal $l_{(i,j)}$ by a component (i,j). By $i+1$ we mean $i+1$, if $1 \leqslant i \leqslant n-1$ and 1 if $i = n$.

$$Sys(F) = \big(K, \{A_{(i,j)}\}_{(i,j)\in K}, C, \{T_{i,j}\}_{(i,j)\in K}\big), \quad \text{where}$$
$$K = \big\{(i,j) \mid 1 \leqslant i \leqslant n, 0 \leqslant j \leqslant 3\big\},$$
$$A_{(i,0)} = \{init_{(i,0)}, false_{(i,0)}\} \quad \text{for } 1 \leqslant i \leqslant n,$$
$$A_{(i,j)} = \{init_{(i,j)}, \text{set-to-1}_{(i,j)}, \text{set-to-0}_{(i,j)}, true_{(i,j)}, false_{(i,j)}\} \quad \text{for } 1 \leqslant i \leqslant n,\ 1 \leqslant j \leqslant 3,$$

[1] The model in [6] is more general, introduces a notion of interaction, which is a subset of a connector and distinguishes between connectors and complete interactions. We are able to show NP-hardness for deadlock detection in interaction systems without the use of complete interactions, so we omit them for ease of notation. Note that this yields a stronger, not weaker result than using complete interactions. Readers who are familiar with interaction systems may simply assume $Comp = \emptyset$ for $Sys(F)$ in Section 3.

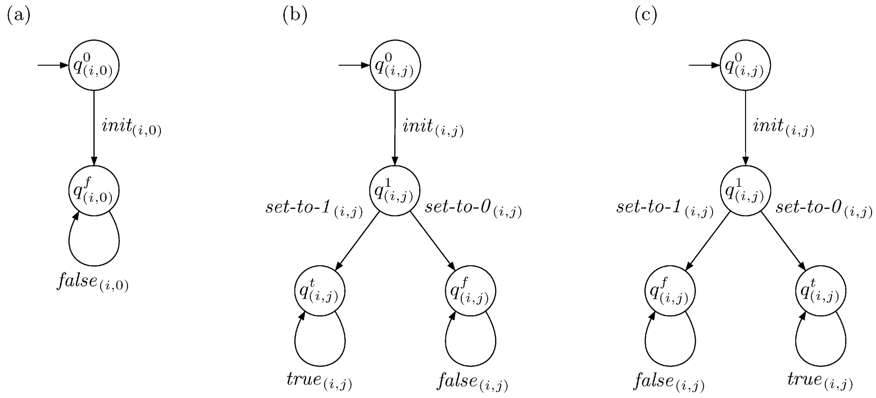

Fig. 1. The $T_{(i,j)}$'s for clause-components (a) and literal-components (b) and (c).

$$\begin{aligned}
C := \big\{ &\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\} \mid 1 \leqslant i \leqslant n\big\} \\
&\cup \big\{\{\text{set-to-1}_{(i_1,j_1)}, \text{set-to-1}_{(i_2,j_2)}, \ldots, \\
&\quad \text{set-to-1}_{(i_a,j_a)}\} \mid \exists \text{ variable } x \text{ that occurs} \\
&\quad \text{in } l_{(i_1,j_1)}, \ldots, l_{(i_a,j_a)} \text{ and only there}\big\} \\
&\cup \big\{\{\text{set-to-0}_{(i_1,j_1)}, \text{set-to-0}_{(i_2,j_2)}, \ldots, \\
&\quad \text{set-to-0}_{(i_a,j_a)}\} \mid \exists \text{variable } x \text{ that occurs} \\
&\quad \text{in } l_{(i_1,j_1)}, \ldots, l_{(i_a,j_a)} \text{ and only there}\big\} \\
&\cup \big\{\{false_{(i,0)}, false_{(i,1)}, false_{(i,2)}, false_{(i,3)}\} \mid \\
&\quad 1 \leqslant i \leqslant n\big\} \\
&\cup \big\{\{true_{(i,j)}, init_{(i+1,0)}\} \mid 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant 3\big\}.
\end{aligned}$$

The local transition systems $T_{(i,0)}$ for $1 \leqslant i \leqslant n$ are given in Fig. 1(a). The local transition systems $T_{(i,j)}$ for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant 3$ and $l_{(i,j)}$ is a positive (respectively, negative) literal are given in Fig. 1(b) (respectively, (c)).

We call components $(i, 0)$ clause-components and components (i, j) where $1 \leqslant j \leqslant 3$ literal-components. For a component (i, j) we call the state $q^f_{(i,j)}$ its *false-state* and, if it exists, the state $q^t_{(i,j)}$ its *true-state*. We call both $q^t_{(i,j)}$ and $q^f_{(i,j)}$ *local final states*. We call a global state $q \in Q$ *global final state*, if all components are in local final states in q.

There is a natural 1-to-1-correspondence between assignments and reachable global final states:

An assignment σ for F corresponds to the global final state $q^{\text{end}} := state(\sigma)$, where all clause-components are in their *false*-states (they have no other local final state) and any literal-component (i, j) that represents a literal of variable x with $\sigma(x) = 1$ ($\sigma(x) = 0$) is in the local final state that is reachable by the set-to-1-action (by the set-to-0-action).

A global final state q^{end} that is in fact reachable starting in q^0 (i.e., all literal-components for the same variable have been set conjointly) corresponds to the assignment $\sigma := ass(q^{\text{end}})$, where for each variable x, $\sigma(x) = 1$ ($\sigma(x) = 0$) if the literal-components in which x occurs are in their local final states that are reached by the set-to-1-action (by the set-to-0-action).

Example 1. Let $F = (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$.

Then, F is satisfiable, namely $\sigma(F) = 1$ for $\sigma(x_1) = 1$, $\sigma(x_2) = 1$, $\sigma(x_3) = 0$.

Consider the corresponding interaction system $Sys(F) = (K, \{A_i\}_{i\in K}, C, \{T_i\}_{i\in K})$, where $K = \{(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), \ldots, (3, 3)\}$ and the port sets $\{A_i\}_{i\in K}$ as well as the local transition systems $\{T_i\}_{i\in K}$ can be seen from Fig. 2.

$$\begin{aligned}
C := \big\{ &\{init_{(2,0)}, init_{(1,1)}, init_{(1,2)}, init_{(1,3)}\}, \\
&\{init_{(3,0)}, init_{(2,1)}, init_{(2,2)}, init_{(2,3)}\}, \\
&\{init_{(1,0)}, init_{(3,1)}, init_{(3,2)}, init_{(3,3)}\}\big\} \\
\cup \big\{ &\{\text{set-to-1}_{(1,1)}, \text{set-to-1}_{(2,1)}, \text{set-to-1}_{(3,1)}\}, \\
&\{\text{set-to-1}_{(1,2)}, \text{set-to-1}_{(2,2)}, \text{set-to-1}_{(3,2)}\}, \\
&\{\text{set-to-1}_{(1,3)}, \text{set-to-1}_{(2,3)}, \text{set-to-1}_{(3,3)}\}\big\} \\
\cup \big\{ &\{\text{set-to-0}_{(1,1)}, \text{set-to-0}_{(2,1)}, \text{set-to-0}_{(3,1)}\}, \\
&\{\text{set-to-0}_{(1,2)}, \text{set-to-0}_{(2,2)}, \text{set-to-0}_{(3,2)}\}, \\
&\{\text{set-to-0}_{(1,3)}, \text{set-to-0}_{(2,3)}, \text{set-to-0}_{(3,3)}\}\big\} \\
\cup \big\{ &\{false_{(1,0)}, false_{(1,1)}, false_{(1,2)}, false_{(1,3)}\}, \\
&\{false_{(2,0)}, false_{(2,1)}, false_{(2,2)}, false_{(2,3)}\}, \\
&\{false_{(3,0)}, false_{(3,1)}, false_{(3,2)}, false_{(3,3)}\}\big\} \\
\cup \big\{ &\{true_{(1,1)}, init_{(2,0)}\}, \{true_{(1,2)}, init_{(2,0)}\}, \\
&\{true_{(1,3)}, init_{(2,0)}\}, \{true_{(2,1)}, init_{(3,0)}\}, \\
&\{true_{(2,2)}, init_{(3,0)}\}, \{true_{(2,3)}, init_{(3,0)}\}, \\
&\{true_{(3,1)}, init_{(1,0)}\}, \{true_{(3,2)}, init_{(1,0)}\}, \\
&\{true_{(3,3)}, init_{(1,0)}\}\big\},
\end{aligned}$$

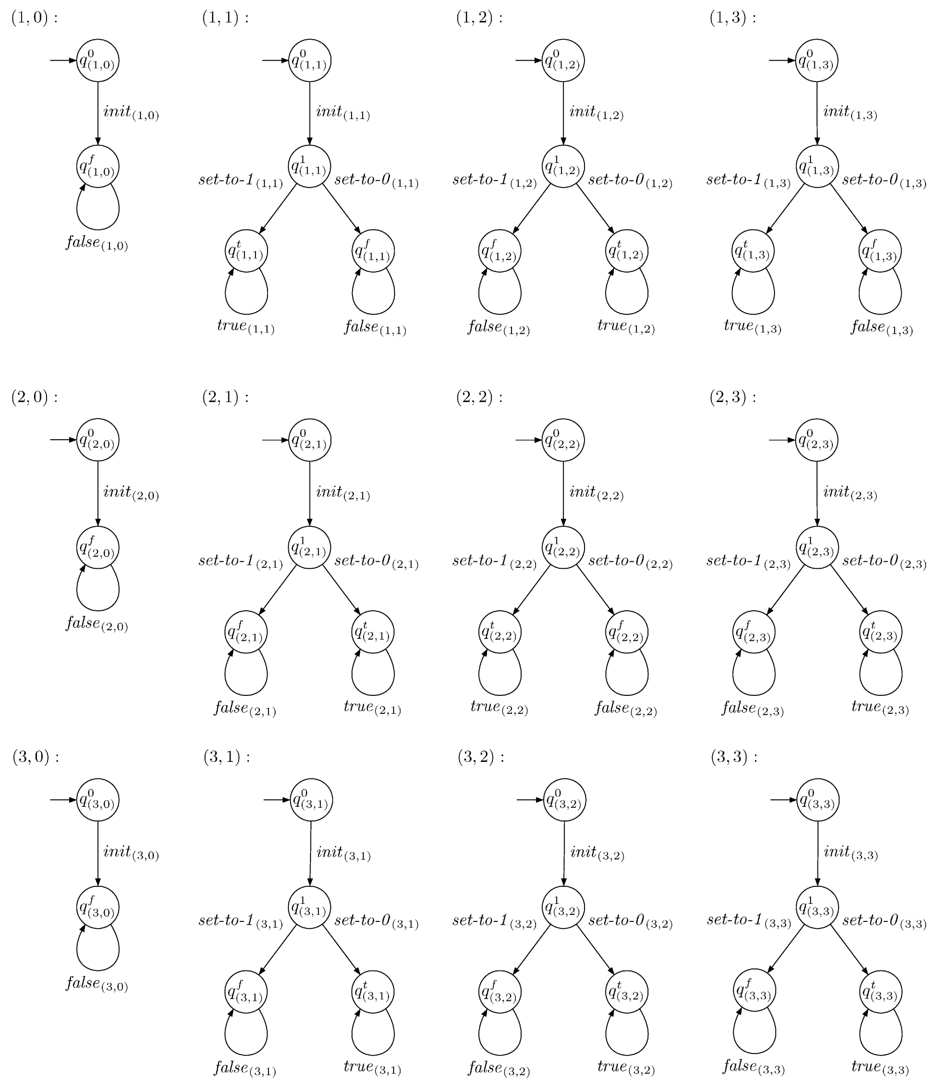

Fig. 2. The local transition systems $\{T_{(i,j)}\}_{(i,j)\in K}$ for Example 1.

$$q^0 = \big(q^0_{(1,0)}, q^0_{(1,1)}, q^0_{(1,2)}, q^0_{(1,3)}, q^0_{(2,0)}, q^0_{(2,1)}, q^0_{(2,2)}, q^0_{(2,3)}, q^0_{(3,0)}, q^0_{(3,1)}, q^0_{(3,2)}, q^0_{(3,3)}\big).$$

As said above, F is satisfiable by σ, so we will show that $Sys(F)$ can reach the global final state $state(\sigma)$, where K is in deadlock:

We subsequently perform the interactions $\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\}$ for all $1 \leqslant i \leqslant 3$.

Then, the clause-components $(i,0)$ are in their states $q^f_{(i,0)}$ and the literal-components (i,j) are in their states $q^1_{(i,j)}$.

Now, we perform:

$\{\text{set-to-1}_{(1,1)}, \text{set-to-1}_{(2,1)}, \text{set-to-1}_{(3,1)}\}$,

$\{\text{set-to-1}_{(1,2)}, \text{set-to-1}_{(2,2)}, \text{set-to-1}_{(3,2)}\}$ and

$\{\text{set-to-0}_{(1,3)}, \text{set-to-0}_{(2,3)}, \text{set-to-0}_{(3,3)}\}$.

Then, K is in deadlock in the global state

$$q^{\text{end}} = \big(q^f_{(1,0)}, q^t_{(1,1)}, q^f_{(1,2)}, q^f_{(1,3)}, q^f_{(2,0)}, q^f_{(2,1)}, q^t_{(2,2)}, q^t_{(2,3)}, q^f_{(3,0)}, q^f_{(3,1)}, q^f_{(3,2)}, q^t_{(3,3)}\big).$$

The global deadlock situation is displayed in Fig. 3, where the nodes (i,j) represent the components (not their local states) and an edge from node (i_1,j_1) to (i_2,j_2) means that (i_1,j_1) waits for (i_2,j_2).

Polynomiality of the reduction. There is no critical blow-up in notation when we go from F to $Sys(F)$. The four transition systems we introduce for each clause are of constant size. Also, the set-to-1- and set-to-0-connectors have an overall size which is linear in the

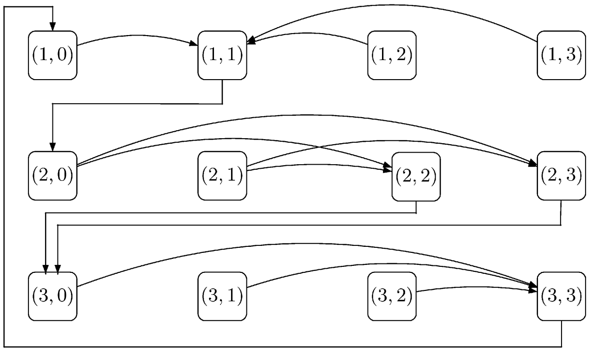

Fig. 3. A graphical representation of the global deadlock in q^{end} in Example 1.

number of literals in F and the other $(5n)$ connectors in C are of constant size.

Remark 1. D is in local deadlock in a reachable state q and $(i, j) \in D \Rightarrow (i, j)$ is in a local final state.

Assume that $(i, 0)$ $(1 \leqslant i \leqslant n)$ is part of a deadlock $D \subseteq K$ and in its local non-final state $q^0_{(i,0)}$. Obviously in any case, the enabled $init_{(i,0)}$-action can be performed together with the $init_{(i-1,j)}$-actions of the corresponding literal-components, as those cannot have left their starting states, so $(i, 0)$ cannot be part of a deadlock.

Assume that (i, j) $(1 \leqslant i \leqslant n, 1 \leqslant j \leqslant 3)$ is part of a deadlock $D \subseteq K$ and in one of its local non-final states:

If (i, j) is in $q^0_{(i,j)}$, then the $\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\}$-interaction can still be performed because the actions $init_{(i,j)}$ $(1 \leqslant j \leqslant 3)$ occur in no other connector and the action $init_{(i+1,0)}$ occurs in other connectors $\{true_{(i,j)}, init_{(i+1,0)}\}$ but only together with the true-actions of the discussed components (i, j) which they do not offer until they have left their starting states which is not the case as we assumed that (i, j) is in $q^0_{(i,j)}$. So (i, j) cannot be part of a deadlock and, in particular, (i, j) can still proceed to $q^1_{(i,j)}$.

If (i, j) is in $q^1_{(i,j)}$, then the set-to-1- or set-to-0-actions can still be performed in the future, because no other literal-component of the same variable can have reached a local final state, because they can only transition conjointly (see definition of C). Also, any of these literal-components can proceed to $q^1_{(i,j)}$ as explained above, if it is not there already.

So (i, j) can still perform some action in the future and thus cannot be part of a deadlock.

So (i, j) must be in a local final state.

Lemma 1. $(Sys(F) \in GDIS) \Leftrightarrow (Sys(F) \in LDIS)$.

Proof. $(\Rightarrow)$ By definition, a global deadlock is a special case of a local deadlock.

$(\Leftarrow)$ (1) Let q be a reachable state in $Sys(F)$, s.t. $D \subseteq K$ is in local deadlock in q. Then a literal-component (i, j) $(1 \leqslant j \leqslant 3)$ participates in D (because the clause-components do not communicate with each other directly).

(2) Due to Remark 1, (i, j) must be in a final state. We show that at least one of the literal-components of clause i must be in its *true*-state: Assume that (i, j) is in $q^f_{(i,j)}$ (else we are done). Then, $ea(q^f_{(i,j)}) = \{false_{(i,j)}\}$, which occurs in the connector $\{false_{(i,0)}, false_{(i,1)}, false_{(i,2)}, false_{(i,3)}\}$. Even if $(i, 0) \in D$, $(i, 0)$ would have to be in its local final state $q^f_{(i,0)}$, so (i, j) would not wait for $(i, 0)$. Hence, one of the literal-components of clause i must participate in D and be in a final state (due to Remark 1) where it does not offer the false action, i.e., its *true*-state.

(3) The literal-component of clause i, which is in its *true*-state can only wait for the clause-component $(i+1, 0)$. So we have $(i + 1, 0) \in D$ and $(i + 1, 0)$ (due to Remark 1) has to be in its only local final state, i.e. its *false*-state.

(4) As $(i + 1, 0) \in D$ offers $false_{(i+1,0)}$, at least one of the literal-components of clause $i + 1$ has to be in D and in its *true*-state. From here, we apply induction by going to (3) and conclude the same for all clauses. $\square$

Corollary 1 *(out of $\Leftarrow$). If $Sys(F)$ is in global deadlock, at least one of the literal-components of each clause is in its true-state.*

Lemma 2. $(F$ *is satisfiable*$) \Leftrightarrow (Sys(F) \in GDIS)$.

Proof. $(\Rightarrow)$ Let $F = k_1 \wedge \cdots \wedge k_n$ with $k_i = (l_{(i,1)} \vee l_{(i,2)} \vee l_{(i,3)})$ be a satisfiable 3-CNF formula and let $\sigma(F) = 1$ for an assignment σ.

The starting state of $Sys(F)$ is $q^0 := (q^0_{(1,0)}, q^0_{(1,1)}, q^0_{(1,2)}, q^0_{(1,3)}, q^0_{(2,0)}, \ldots, q^0_{(n,3)})$. Let $Sys(F)$ perform the following transitions:

(1) For all $1 \leqslant i \leqslant n$ perform the interactions $\{init_{(i+1,0)}, init_{(i,1)}, init_{(i,2)}, init_{(i,3)}\}$. Then all clause-components $(i, 0)$ $(1 \leqslant i \leqslant n)$ are in their *false*-states $q^f_{(i,0)}$ and all literal-components (i, j), $\forall 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant 3$, are in their states $q^1_{(i,j)}$.

(2) Let x be a variable that occurs in F at the positions $(i_1, j_1), (i_2, j_2), \ldots, (i_a, j_a)$ (and only there), and let $\sigma(x) = 1$ (or $\sigma(x) = 0$, respectively).

Then perform the interaction $\{$set-to-$1_{(i_1,j_1)}$, set-to-$1_{(i_2,j_2)}, \ldots,$ set-to-$1_{(i_a,j_a)}\}$ (or $\{$set-to-$0_{(i_1,j_1)}$, set-to-$0_{(i_2,j_2)}, \ldots,$ set-to-$0_{(i_a,j_a)}\}$, respectively).

After having performed the corresponding interaction for each variable that occurs in F we reached the global final state $q^{\text{end}} = state(\sigma)$ that we described above.

As $\sigma(F) = 1$ we have $\sigma(k_i) = 1\ \forall 1 \leqslant i \leqslant n$, i.e., in each clause there is at least one literal that evaluates to 1 under σ. This means there is at least one positive literal $l_{(i,j)} = x$ with $\sigma(x) = 1$ or a negative literal $l_{(i,j)} = \bar{x}$ with $\sigma(x) = 0$. In both cases the corresponding transition system $T_{(i,j)}$ has reached its local state $q^t_{(i,j)}$ (cf. Fig. 1 (b) and (c)).

Hence, we have $\forall 1 \leqslant i \leqslant n$, $q^{\text{end}}(i, 0) = q^f_{(i,0)}$ and $ea(q^f_{(i,0)}) = \{false_{(i,0)}\}$.

Furthermore, $\forall 1 \leqslant i \leqslant n\ \exists j \in \{1, 2, 3\}$, s.t. $q^{\text{end}}(i, j) = q^t_{(i,j)}$ and $ea(q^t_{(i,j)}) = \{true_{(i,j)}\}$.

Obviously, $Sys(F)$ is in global deadlock in q^{end} (or in other words $D = K$ is in deadlock in q^{end} in $Sys(F)$), as every clause-component $(i, 0)$ waits for at least one of its literal-components $(i, 1), (i, 2), (i, 3)$. Those literal-components in $(i, 1), (i, 2), (i, 3)$ that are in their q^f-states, also wait for those that are in their q^t-states and those that are in their q^t-states wait for the clause-component $(i + 1, 0)$. Hence, we observe a cyclic waiting over all clauses (cf. Example 1, Fig. 3), including all components. The global deadlock is also a local deadlock.

$(\Leftarrow)$ $F \in GDIS$ means that there is a reachable global state, where K is in deadlock.

By Corollary 1 we know that at least one component of every clause must be in its true state.

Due to the one-to-one correspondence of literal-components to literals and the fact that all occurrences of a variable x are consistently set to a value $\in \{0, 1\}$ and the fact that in each clause at least one literal evaluates to "true", we may conclude the existence of a satisfying assignment σ. □

So by now we know that the existence of a deadlock D implies that F is satisfiable. Yet, it is still possible that some variables have not yet been set to 0 or 1, i.e., the corresponding literal-components $(\tilde{i}, \tilde{j})$ are not yet in their final states, so the deadlock D would not be global. It is however quite obvious, that we still may perform interactions such that these $(\tilde{i}, \tilde{j})$ finally reach local final states. We call the thus reached state q' and in q', $D = K$ is in global deadlock, because the $(\tilde{i}, \tilde{j})$, wait for components that participate in the cyclic waiting, no matter if $q'(\tilde{i}, \tilde{j}) = q^t_{(\tilde{i},\tilde{j})}$ or $q'(\tilde{i}, \tilde{j}) = q^f_{(\tilde{i},\tilde{j})}$. So the existence of a local deadlock implies that F is satisfiable as well as the existence of a global deadlock.

4. Conclusion and related work

We showed that the questions of local and global deadlock are NP-hard for interaction systems, even without the use of complete interactions. This yields a motivation for establishing sufficient conditions for deadlock-freedom that can be tested in polynomial time. One such condition has already been presented in [9] and we are presently working on an enhanced version that covers a larger set of systems without raising the given time bounds. Both approaches try to tackle the problem by a combination of two ideas:

First, we perform state space analyses in subsystems (i.e., systems that are gained by projecting the whole system on subsets of K) instead of performing a global state space analysis. When projecting to subsets $K' \subseteq K$ we lose information about the non-observed components $K \setminus K'$. We are however able to handle this loss of information in such a way that it yields an over-approximation of the reachable state space.

Second, we use a locally (in K') applicable sufficient condition for deadlock-freedom (which is again an over-approximation) to verify deadlock-freedom for the thus computed set of reachable states of subsystems.

If the algorithm succeeds for a particular system, we have shown that no state that includes a local deadlock is reachable. However, if the algorithm fails, we have gained no information about whether the system contains a deadlock or not.

As far as NP-completeness of the discussed problems is concerned, there seems to be no trivial way to show that the problem is in NP, because even if we guess a deadlock state, it might take an exponentially long transition sequence to reach it from the starting state. (Which means that verifying the states reachability might take exponential time.) For the related model of Parallel Processes, Ladkin and Simons showed in [8] that deadlock-detection is NP-hard. A sufficient condition for liveness in interaction systems is given in [2]. A different approach to ensure deadlock-freedom and progress is given in [3].

References

[1] A. Basu, M. Bozga, J. Sifakis, Modeling heterogeneous real-time components in BIP, Invited Lecture, SEFM, 2006.

[2] G. Gössler, S. Graf, M. Majster-Cederbaum, M. Martens, J. Sifakis, An approach to modelling and verification of component based systems, in: Theory and Practice of Computer Science, Lecture Notes in Comput. Sci., vol. 4362.

[3] G. Gössler, S. Graf, M. Majster-Cederbaum, M. Martens, J. Sifakis, Ensuring properties of interaction systems, in: Program Analysis and Compilation, Theory and Practice, Lecture Notes in Comput. Sci., vol. 4444.

[4] S. Cook, The complexity of theorem proving procedures, in: Proceedings Third Annual ACM Symposium on Theory of Computing, 1971, pp. 151–158.

[5] M.R. Garey, D.S. Johnson, Computers and Intractability, a Guide to the Theory of NP-Completeness, W.H. Freeman, New York, 1979.

[6] G. Gössler, J. Sifakis, Component-based construction of deadlock-free systems, in: FSTTCS, in: Lecture Notes in Comput. Sci., vol. 2914, Springer, 2003, pp. 420–433.

[7] G. Gössler, J. Sifakis, Composition for component-based modeling, Sci. Comput. Program. 55 (1–3) (2005) 161–183.

[8] P. Ladkin, B. Simons, Compile-time analysis of communicating processes, in: Proceedings of the 1992 International Conference on Supercomputing, ACM Press, 1992, pp. 248–259.

[9] M. Majster-Cederbaum, M. Martens, C. Minnameier, A polynomial-time checkable sufficient condition for deadlock-freedom of component-based systems, in: Theory and Practice of Computer Science, Lecture Notes in Comput. Sci., vol. 4362.

[10] J. Sifakis, Modeling real-time systems, Keynote talk RTSS04.

[11] J. Sifakis, A framework for component-based construction, Extended abstract, in: SEFM, 2005, pp. 293–300.

[12] A.S. Tanenbaum, Modern Operating Systems, second ed., Prentice-Hall, 2001.